Has Perspective a future? (Has Man a future?)

Analogies between questions on the future of Man in the face of the disasters of the indiscriminate use of the science and the technology (during the era of the Cold War, but still of interest today) and some considerations on the future of the perspective (and the descriptive geometry) in the era of the automatons.

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I have been involved in Applications of descriptive geometry over the last thirty years. And after so long time I started to see an historical perspective. This is impossible when you are young: History is a flat wall painting, something to study and to learn by heart, there is not much to understand. But as the years go by you realize that History is alive and that we live within it, as fish in the water of a river that runs away. I can therefore look back and also forward, looking in the direction of the stream. And I can describe what I think I'm discerning. But most of all I can, from my point of view, try to change the course, moving myself by strengths of arms to the one side or to the other.

Thirty years ago the descriptive geometry was orphan. The mathematicians had given it up a short time before, not only because of a loss of interest for the scientific speculations on this subject, but also because of the consequent abandonment of the respective chairs.
Now, anyone who studies descriptive geometry knows that the theory of this science, in that time, searched its own demonstration on the blackboard. The classroom blackboards of Polytechnics and Universities were the battleground where the ideas and their application results were put to the test. Of course, it was also possible to construct a fine drawing with a hard pencil or a pen, but the blackboard … the freehand drawn stroke of the chalk, together with, in a loud voice, the effort of the thought that justified its progressing, was the best that could be hoped for. In this flight without net, from one trapeze to another, was the essence, the soul of the descriptive geometry. If guided by lucid reasoning and without uncertainty, the exercise could finish in a figure of high aesthetic value. Otherwise … the fall (fall, which was always lying in wait). I realize that this circus metaphor may appear out of place, since we are talking about science. But it is not, since the descriptive geometry is a very particular science, always compromised with the art and made of thought and images (and therefore of metaphors, too) in one only body. The descriptive geometry cannot be made without images, and the images, which guide the project from the invention to the realization, cannot be done without the descriptive geometry. Of course, when I say descriptive geometry I'm not only thinking about Gaspard Monge. I'm not thinking about him more than I'm thinking about Vitruvio, about Piero, about Desarges and his friend Bosse, about Taylor, about Frézier and about all the other artists and scientists that dealt with this science before Monge gave it a name, which History engraved forever[1]. Well, the reason for the abandonment, on behalf of the mathematicians, can perhaps be found in its particular character: in a steady progress towards a sublime abstraction, the mathematics has deviated from the arts. Maybe the mathematicians, in the epoch that we are talking about, did not know how to draw anymore. And it is funny to observe how a great mathematician, who is a powerful champion of the analogical representation and of the models, namely René Thom, speaks with ill-concealed contempt about the descriptive geometry (in this case in the Mongian accepted meaning), as of an obsolete science[2].

[1] My opinion on Monge's work, bitterly challenged by some colleagues of mine, is better stated and, above all, explained in the volume *Scienza della Rappresentazione* (Rome 1992), written together with Mario Docci, and, more recently, in the short essay *La vita e l'opera di Gaspard Monge: critica di una apologia* published in the review *XY, dimensioni del disegno*, X, 27-28, May-December 1996.

[2] The modernist movement has, in part, renewed the teaching of the mathematics that in France was characterized by a certain kind of sclerosis, due to the existence of the exams of the Grandes Écoles. From the very beginning the students were educated to be able to pass these exams: the programmes were determined once and for all and were

And what about the architects? What did the architects do in that period of transition? The architects learned from the mathematicians, in other words they gathered their heritage. In Rome Orseolo Fasolo received the baton from Enrico Bompiani (through Maria Luisa Ganassini, engineer); in Florence Ugo Saccardi received it from Nello Baroni in the hereditary line of Luigi Campedelli; and so in Naples and in other Universities, too. Of course, each heir added a little of his own, as it is right and useful, also because none of these Masters, that I have known and know personally, and with whom I often have spoken about this problem, none of them pretended to reach that logic rigour, that language, that intensity of references that was characteristic to the mathematical exposition. And all of them, maybe, felt a little at fault, too, for that. Mistaking, I could say today with hindsight, because what the descriptive geometry lost in science, it regained in art. You only need to glance through the books that these Masters have produced, and to confront them with the last works of the mathematicians (a bit listless, to tell the truth) to realize it [fig. 1 e fig. 2].

In this context, and in this atmosphere of transition, enters the picture that I would rapidly like to outline, the one of the perspective at the beginning of its apogee, in the architectural schools of thirty years ago. The hand-over of the descriptive geometry, from the mathematicians to the architects, marked, in fact, the beginning of a revision and development process of the perspective studies, as it was to be expected, considering the return, into the riverbed of the discipline, of that artistic component that the mathematicians had neglected. It would not have been fair, nevertheless, to accuse the mathematicians of insensitivity to these aspects of the architectural studies, since it was exactly Francesco Severi, mathematician of very high quality, who developed the teaching programmes of the descriptive geometry at the foundation of the Roman school and who promoted, therefore, the renewal that I would like to analyse[3].

First of all the architects recovered the relationship with the noble and antique history of the descriptive geometry. As a matter of fact after Gino Loria[4], who wrote in 1921, nobody had any interest in the tradition anymore, except for Francesco Severe who, in his opening lectures, introductions and notes, never forgot to remember the origins[5]. Luigi Vagnetti and Orseolo Fasolo as the first, I believe, started a methodical rereading of the sources, from the Latin authors (in-depth studies carried out by Rocco Sinisgalli), to Monge and to the writers of treatises of the 19th Century, like Fiedler (studies carried out, in particular, by Orseolo Fasolo). Many important works have followed and still follow, for instance, in the Neapolitan school.

Secondly, the architects take the credit for having given new impetus to studies that were nearly or totally forgotten, like the theory of the shades and the chiaroscuro. The mathematicians, in fact, were not very interested in these theories and, most of all, they were not able to apply the related techniques.

As far as the perspective concerns, in particular, the nuisance of the mathematicians clearly appears through their reluctance to reassemble the perspective of the artists, most of all made of practical rules, and the central projection, namely the heritage of Brook Taylor handed over to the artists[6].

The two methods can be found in different chapters[7] in the last works of the mathematicians, as if they had different theoretical premises and finalities, and there are also other funny representations totally taken

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practically everlasting. For instance, the descriptive geometry was taught because Monge, the creator of the École Polytechnique, had set it up. From R. Thom, *Parabole e catastrofi, intervista su matematica scienza e filosofia*, care of Giulio Girello and Simona Morini, Milan 1980.

[3] The lessons of Severi and, as a consequence, his programmatic aims, have reached us unimpaired. See F. Severi, *Applicazioni di geometria descrittiva, lezioni del prof. F. Severi*, collected by Michele Campanella (II° Year of the Faculty), Faculty of Architecture of Rome, A. XIV E.F. See also: R. Migliari, *L'insegnamento della geometria descrittiva e delle sue applicazioni*, in *La Facoltà di Architettura dell'Università di Roma "La Sapienza" dalle origini al duemila, Discipline, Docenti, Studenti*, care of Vittorio Franchetti Pardo, Rome 2001.

[4] See G. Loria, *Storia della geometria descrittiva, dalle origini sino ai giorni nostri*, Milan 1921.

[5] See F. Severi, *op. cit.*

[6] See B. Taylor, *Linear Perspective Or A New Method Of Representing Justly All Manner Of Objects As They Appear To The Eye In All Situations, A Work Necessary For All Painters, Architects Etc. To Judge Of, And Regulate Designs By*, London 1715; and the second edition, better known: B. Taylor, *New Principles of Linear Perspective or the Art of Designing on a Plane the Representations of all Sorts of Objects, in a More General and Simple Method Than Has Been Done Before*, London 1719.

out of the necessity to observe the represented object. So, for example, Enrico Bompiani designs a parallelepiped in a way that the image is placed entirely out of the distance circle and it is, therefore, indecipherable[8] (see fig. 2). In other words, Bompiani points out the possibility to make use of the central projection in the production of perspective images, but he does not give us a clear example.
Well then, the architects faced the task to re-unify the two kinds of perspectives, enlightening the relations with the homology[9] and developing the possible applications.
This operation has a historical value of great importance; it is the fall of the Berlin Wall, the end of an epoch of absurd discriminations, but also and above all the end of an incorrect conception. It is worth, therefore, briefly to dwell upon this point.
As it is common knowledge, the perspective has an old history that places in the first Renaissance its "rediscovery" on behalf of artists-scientists, like Piero della Francesca and Leon Battista Alberti. In these first periods of its existence, the perspective is "one" and the method of the orthogonal projections is bent in order to help and to validate the perspective construction. Furthermore, in the writers of treatises is already very strong the effort to free themselves from the use of auxiliary projections, in order to work only on the perspective picture and within a clearly-defined area, the one that contains the image, likely to permit the artist to construct and to control the big trompe l'oeil perspectives. So it is throughout the eighteenth century and while the relations of the perspective with the incommensurable distances represented by vanishing points and lines were discovered and consolidated.
Barnaba Brisson, editor of Monge's work, introduces the perspective[10], in the fourth edition of Géométrie Déscriptive (1820), with the title *Théorie des ombres et de la perspective.* And then there is a reversal of values. Indeed, the method of the orthogonal projections allows the construction of the perspective image using points, without resorting to other and more complex considerations (as Piero on the other hand already knew). And it therefore appears needless to recover and to develop the knowledges gathered until then. Actually, only the vanishing point (*point de concours)* is retained in the short Mongian treatise with the aim to improve the accuracy of the constructions. And the perspective is delivered back to the artists as a simple application of the projection method[11].
So, to be clearer, with the Mongian treatise the perspective looses its dignity as method of representation, able to autonomously describe the forms of the space and their reciprocal relations, to become a simple product of the method of Monge. The mathematicians, for their part, continue to study and to develop the perspective with different purposes (luckily) and the perspective assumes the very pompous name "central projection", in contraposition to the "parallel projection" which generates planes, elevations and axonometries.

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[7] One example among all others: Gino Fano, *Lezioni di geometria descrittiva date nel R. Politecnico di Torino*, Turin 1925 (third edition). In this essay, which maybe is the best and the most complete of an Italian author, the central projection is explained in the second chapter of the methods of representation, immediately after the elements of the projective geometry and before the orthogonal projections, whereas the perspective, even if included among the methods, is illustrated in the fifth chapter, after the elements of the theory of the shades.

[8] I owe this observation to Maria Grazia Fioriti, my fellow student during the period 1967-71 at the Faculty of Architecture of the "Sapienza" University in Rome.

[9] See O. Fasolo, *L'Omologia e le sue applicazioni alla rappresentazione progettuale e tecnica dell'architettura, Quaderni di Applicazioni della geometria descrittiva*, n. 4, Rome 1989.

[10] The perspective of Monge is founded on the points and the accuracy of the result on the number of these points, whilst we know that to a larger number of operations there corresponds, inevitably, a wider propagation of the error. It is however true that the mathematicians of the nineteenth-century seem indifferent to this problem: just think about the abused construction of the Steiner ellipse given the conjugate diameters, which, beyond the conceptual value, produces forms as much distorted as the number of points considered is bigger and that humiliates, therefore, the symmetry endowments of the curve.

[11] The *Géométrie Descriptive* imposes, in reality, two ways to draw and to study the perspective: the first, practical and addressed to the "artists" in the illuminist sense of the word, namely the craftsmen, in the *incipit* spirit of the Mongian work; the second, more cultured and refined, prerogative of the mathematicians, *humus* in which the new conquests of the descriptive geometry can grow. And it was really a radical change, because, as I already mentioned talking about Piero, the artists' efforts to work only on the picture and on the wall, without using auxiliary projections, just think about Desargues, as one only example, went for the benefit of the most abstract theory. Whilst the method at the artists disposal, with Monge, regressed to infancy.

At the end of the nineteenth century, indeed, the mathematician Wilhelm Fiedler tries to heal this rift. He traces the representation methods back to one only origin and he sees in the central projection the theory within which the perspective takes its place as general case and the parallel projections as particular case[12].
It's interesting to notice that Fiedler also made use of the perspective to re-introduce the homology of the solid systems "considered as the theory of the methods of the art of modelling"[13]; I will come back to this at the end of this work, also because it reveals an amazing topicality. Unfortunately, though, Fiedler's position is very advanced for the time and therefore not taken up again. Gino Fano[14], for instance, even if quoting Fiedler, among his first references, immediately after Monge, did not at all take up his logical sequence of the subjects, which remains traditional, but only the use that he makes of the homological relations in the solution of representation problems[15].
This is how, in the school of thirty years ago, still was alive the unfair distinction between the central projection (noble and abstract perspective) and the perspective or method of the architects (empirical perspective and a little uninspired, too).
So, in the moment when the mathematicians left the descriptive geometry into the hands of the architects, the perspective recovers its generality of method and its true nature as "general case" e synthesis of the representations, that Vitruvio already recognizes it, and that, with Fiedler, rises to be a theoretical foundation.
As regards the theory of the shades and the chiaroscuro, then, the mathematicians only constructed the contour of the shades, leaving that the background was carried out all in black and dark grey, without differentiating the degree of luminous intensity and even without distinguishing between proper shade and brought shade, describing so an universe of desperate contrasts, a lunar landscape that nothing had to do with the "masterly game" of LeCorbusian memory. And yet, what a lot of studies of high theoretical and artistic value were dedicated to this subject in the late 19th [16] and the early 20th century[17]! Almost as if the above said theory was marginal in comparison with the disciplinary corpus. The theory of the shades and the chiaroscuro is instead part of the perspective, since it teaches to simulate those phenomenons that allow the perception of the depth (aerial perspective, for instance) and that add their effects to those, purely geometrical, of the linear perspective[18].
But let's come back to our history.
So, thirty years ago the perspective was at the centre of the studies on the representation thanks to the shift from the disciplinary contest of the mathematics to that of the architecture. But I believe, too, because of reasons different from the contribution of the architects. I believe, in other words, that the centrality and the topicality of the perspective depend on some immanent values. The first of these values is the Man; the second is his conception of the infinity.
The perspective is the only representation method that introduces the center of projection, or, simply, the eye of who observes, into the represented space. On the contrary, the parallel projections conventionally define that the projection center is placed at an "infinite" distance and, therefore, always and only out of the space that they represent. But an observer placed at an "infinite" distance is a contradiction in terms, since the infinity is a metaphysical category[19], which has nothing to do with the drawings and with the sight. It would be much better to state that the visual pyramid is, hypothesis assumed arbitrarily, a

[12] See G. Fiedler, *Trattato di geometria descrittiva*, Florence 1874.
[13] Fiedler writes, in the preface of his essay : " … That peculiar reciprocal position that the two figures have at the moment of the representation is called *perspective position*. The theory of the plane representations according to these basic principles, we will call them *doctrine of the central projection*; the perspective is part of it and, as a special case, the *parallel projection*, oblique or orthogonal". op. cit. page. 3.
[14] G. Fano, op. cit.
[15] This interpenetration of the projective geometry with the descriptive geometry is mainly owed to W. Fiedler" - *Die darstellende geometrie in organischer Verbindung mit der Geometrie der Lage*, 3 volume (1871)". G. Fano, op. cit. pag. 6.
[16] See D. Tessari, *La teoria delle ombre e del chiaro-scuro*, Turin 1880
[17] See J. J. Pillet, *Traité de perspective linéaire précédé du Tracé des ombres usuelles (Rayon a 45 degrés) et du Rendu dans le dessin d'architecture et dans le dessin de machines*, Paris 1921.
[18] This concept is already very clear in the Mongian description, even with the limits that I mentioned.
[19] See P. Odifreddi, *Le menzogne di Ulisse*, *l'avventura della logica da Parmenide a Amartya Sen*, Milan 2004.

parallelepiped, and to start constructing a conventional drawing, which does not have much to do with the real form of the objects that it represents.
Well, in short, the parallel projections are the true "symbolic form", considering their relation, completely abstract, with the space that they represent.
The perspective, on the contrary, has an operational relationship with the space and it sets man into relation with the objects that he has around himself, like with those he cannot reach. And so as the perspective, understood as method, includes the observer, namely the man, the perspective, this time understood as image, incorporates the man, namely the observer.
A confirmation of this?
Any object represented in the orthogonal projections is devoid of size, at least until a measure is given, namely a dimension, or a term of comparison; or rather until the Meter that is kept in Sèvres will not be reproduced together with the space[20].
All this is not necessary with the perspective. It is enough to know that who observes is a man. If the horizon cuts the represented object at a third of its height, I know for sure that the object is three-men high. The perspective, then, measures the space with the man and it furnishes an immediate and instinctive perception of the dimension even without metric indications.
But, as I said, there is something more.
Orseolo Fasolo loved to say that the perspective allows treating the infinity in finite terms.
As a matter of fact, who uses the vanishing points as images of the directions and the lyings of the space, and, for instance, constructs the intersection of two planes drawing two straight-lines on a peace of paper, is doing exactly this: he makes use of images of the infinite to resolve a finite problem, and he uses two strokes on the paper to visibly reason on what otherwise is invisible and not knowable. I don't want now to take up an argument that risks to take us too far: the infinite and the incommensurable and the logic of the conventions currently used in matters of "improper" entities[21], I only want to say that the question deserves attention, also from who is better equipped than me in logical-mathematical and philosophical knowledge.
But here I wanted to mention these two aspects of the perspective because they have become, with the interactive dynamic models, of great interest today,.
And the students? How did our students experience that happy period? I remember a winter afternoon, a low and golden light that ran through the classroom and made the chalk powder glitter, and a one year older fellow student who, bent over a big drawing of the "Tourette", patiently removes several layers of halftone screens applied onto it, in order to correct an error made when constructing the contour of the shade. This was the moment of the trapeze artist's fall, since, to the extemporaneous and temerarious exercise of the teacher corresponds the patient exercise of the student, on glossy paper, with the pencil at first and the Indian ink, later. Each stroke, each gesture was the fruit of a patient meditation, which develops in a toilsome and slow proceeding, guided by the definite laws of the geometry. We students, we called them "rules". The perspective was that one "made with the rules". Each stroke of the pen was guided by the tenuous bond with the infinite, by means of the vanishing point, pierced by a pin on which the drawing rule leaned. And each vanishing point was tied to the others by the measure of an angle. And the Measure, of the forms and the light, governs that world of abstractions that became as-if-true in a bright image. All this implied a certain and complete knowledge of the represented space, of its inner relations and of its relationship with the man who was observing it. And, not at random, the perspective was the final synthesis of a path of discoveries and of knowledges.

The moment has arrived to ask the question that is at the center of this brief inquiry: has perspective a future?
Bertrand Russell pondered, in 1961, on the risks of a nuclear holocaust. The cold war is over since quite a while, but the dangers are still there and nowadays maybe even more difficult to prevent. But I have to

[20] Consider, for instance, Auguste Choisy's famous axonometries in *Art de bâtir chez …* (*les romains, les byzantins*) …, which always give the associated metric scale.
[21] I discussed this subject in the essay *La prospettiva e l'infinito*, in "*Disegnare, idee immagini*", Anno VI, n. 11, Dicember 1995.

explain what all this has to do with the perspective: well, I confess, maybe this is only one of those flashes that sometimes, suddenly, and unintentionally, explodes in the mind. While I wrote the above pages, I had a distant recollection of a book that I read at the time when, being a boy, I studied English. And I started to ask myself what kind of relationship there could be between Russell's famous essay and my own modest work, between the essential themes that he dealt with in his essay and the themes, insignificant comparing to Russell's, that I'm dealing with in my paper. And while Russell's words continued to resound without apparent reason in my head, another voice, low, tried to find out the reason why.

The perspective is, as everybody knows, one of the greatest expressions of Humanism, namely of those values that make mankind worthy to preserve his species on the Earth and able to create a world without wars, "a world of shining beauty", as Russell says[22]. This is the starting point. There would not be any reason to fight against the nuclear war, against all wars, if there was no hope in a future of justice and of "shining beauty". And this word, so simple and so abused, the word "beauty", has not been chosen at random, because it is an aim in itself, so as its fragments, what remains of them, are a reason to hope and a guide to live.

Now, it is banal, or better, obvious, to say that with the end of mankind, the perspective would die, too; and, rather, somebody could rightly smile at such a poor loss in comparison with a nuclear holocaust. But I, like Russell, do not refer to the physical loss of mankind, but to the ethic loss. What frightens Russell the most is not the destruction of mankind, but the moral degeneration that would lead to this end.

In the first pages of his essay Russell describes the slow and toilsome progress of mankind from the dawn of History up to the most elevated conquests of thought and of art. He celebrates, so, the birth of Humanism and he fights to avoid that this civil progress, which has as purpose a "world of shining beauty", should degenerate into an involution without return. From this point of view, our perspective, which is only a very small part of that pure beauty, nowadays runs similar risks. The siege of the machines is suffocating it.

It is not necessary anymore to challenge the fragile constructions that "treat the infinity in finite terms": the noble exercise of the trapeze, the fly of the chalk on the blackboard, does not exist anymore. And it is even not necessary to observe the haze of the horizon, and to imitate the sky-blue of distant mountains, to reproduce the depth of the space. Today all this can be obtained automatically, the linear perspective as well as the aerial, and with as greater true-to-life accuracy as more refined are the algorithms that reproduce the relative physic phenomenons, and as better is their transformation into instructions for the machines. And the sense of wonder that these applications of the information technology produces is so strong, that the attention of the users is turned towards them, instead of towards the purpose to which they are destined, namely the image.

On the Internet, throngs of "3D artists" challenge each other in search of the perfect simulation, under the insignia of photorealism. Nobody cares anymore about the beauty, but everybody searches for the perfection of the means and looks at the capability of who uses it to create images indistinguishable from an ugly photograph of an ugly true world. There is something pathetic and disquieting in this painful effort to give the fantasies, often of romantic taste[23], a more and more true appearance, neglecting totally the quality of the fantasy itself, its poetic contents, its capability of alluding without telling, its beauty (to say it with one word, not any word, but the word used, precisely, by Russell).

Therefore I believe that to be able to save the perspective, we have, today, consciously and tenaciously, to undertake at least two actions. The first consists in recovering the value and the importance of the artistic judgement, appraising and researching, namely, the expressive quality of an image, its capability to stir up emotions and to transfer messages, whatever technique is used.

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[22] "*It is not only what to avoid that great men have shown us. They have shown us also that it is within human power to create a world of shining beauty and transcendent glory. Consider the poets, the composers, the painters, the men whose inward vision has been shown to the world in edifices of majestic splendour. All this country of the imagination might be ours …*" Bertrand Russel, *Has Man a future?*, London 1961, Prologue or Epilogue?

[23] I refer in particular to that style, which today is called "gothic style". The taste, namely, for the dark atmosphere and the stormy skies, for the disquieting characters in horror films and for video games so fashionable among the young. This is exactly the decadent taste that Mario Praz describes and analyses in his essay *La carne, la morte e il diavolo nella letteratura romantica* published in Florence in 1948.

The second consists in recovering and developing all those theorical contents and controversial questions that the history of perspective has handed down to us. And I will now try to give a couple of examples.
I will start with the practicing of the art.
Why do so many artists today rediscover the toilsome and slow preparation of the colours and why do they refuse to use tube colours even if these allow them to paint *en plein air* and consequently to give birth to Impressionism? Why do many of them rediscover the classical drawing and even the methods of the academic composition? Does all this not reveal an incipient insecurity … the desire to seize, in extremis, something that we are losing: the tradition?
The perspective, which is one of the most important expressions of Humanism, can it survive in a society that is dominated by the machines, and how? Is a Humanism of the machines possible and how?
I am not the right person to answer these questions.
Nevertheless I see, all around me, the first efforts crowned with success, for instance in the graphical work (or pictorial? or digital?) of Gabriele Pierluisi. Gabriele has understood that the future is in the metamorphoses of the models[24], not in the antagonistic contraposition of the techniques, new against old, automatic against manual, but, on the contrary, in the free overlapping of the techniques, all brought back under the control of the man and of the artist. Gabriele, just to give one example among the infinite possible experiments that he is performing, he takes a photography and retouches it using the digital technology and he prints it out in large format onto a rigid support; then he starts to paint on it with acrylic colours, and he expresses in this way his world and its architecture [fig. 3 e fig. 4].

So, if we want to save the perspective, I think that, above all, we should focus on these values, instead of hiding in the tower of old rules or, what is worse, in the quagmire of specialized softwares. And I hope, in this overall picture, in a great return of true-to-life drawing, I dream of classrooms with easels and rooms equipped with computers.
When if was a boy I loved to draw, but the *liceo artistico*, the secondary school specializing in art subjects, was considered, in that time, an inopportune choice, because the students were barred from attending some of the Faculties. So I attended the upper secondary school specializing in classical studies, and also some drawing lessons. These lessons were given by Diego Pettinelli[25], who taught at the Academy, who painted and who was an extraordinary wood engraver (xylography). I started with some boring earthenware, which however teaches me, instinctively, what an apparent contour is; then I began to draw plaster busts: Mario, Giulio Cesare and a very dishevelled Beethoven, and from these famous figures I learned, still instinctively, the chiaroscuro. And my brief career as artist manqué closes finally in the Parnassus of the figure, with the Academy models who posed for our small inexpert group in an atmosphere of great solemnity. How I would like, today, that those simple experiences, which can develop in only few lesson hours, were part of the formation of an architect!
It struck me, a few days ago, the answer given by a Chinese colleague to someone who asked him why, in the oriental schools of Architecture, they teach watercolour painting: “Because the watercolour cultivates the aesthetic sense”, he said. It’s true! As it also teaches a well-balanced use of the colour, exactly what is lacking in many of the above-said “3D artists”. We have to spread this answer, extending it, however, from the watercolour painting to the graphical and pictorial techniques, which already were part of the formation of the architects during the first years of our earliest schools.
I always thought that the famous Vetruvian triad was not only a list of “representation methods” but also a procedure. The sequence of the drawings, in fact, is not random and it seems to be connected to the real experience of the building yard: the first step is to mark out the imprint of the building on the ground: the ichnography; then the heights are measured: the orthography; finally we have an overall vision, as a man perceives it: the scenography or the perspective, as you would like to call it.
The perspective, so, has a value of synthesis and of verification. These values are not lost, still today it is necessary to construct a model, before we can generate a perspective of it. And the construction of the

[24] See AA.VV. *Disegno come Modello*, care of R. Migliari, Rome 2004.
[25] Diego Pettinelli, (Matelica 1897 – Rome 1989), a pupil of Adolfo De Carolis, landscape painter and engraver of extraordinary capability, specialized in xylography.

model requires the same knowledge of the inner relationships that was necessary before. But something has profoundly changed.
In the graphical perspective the eye of an artist who drew was tied to the image by the invisible bonds of the geometric projection; today the image arises by means of an automatism. The automaton has no hart and it has no measure, it can be as tiny as an insect or high as a five-storied building. In the perspective as it is today, not at random, this automaton is not called "eye" but "camera".
So pay attention, because the perspective incorporates the observer!
This means that an unwise use of the camera will describe a space as an insect sees it or perhaps a flying gull, and the architecture will lose its human measure.
Now I am torn between the fear of boring you with the obvious[26] and the everyday teaching experience that tells me how much this concept, and the sensibility that comes from it, are difficult to form. But in doubt, here is a little example.
Let's consider the perspective of a building [fig. 5] and let's observe the point in which the horizon cuts the image of it. We can evaluate, even in a brief way, the height of the point in comparison with the building: if it is, as in our example, between the first and the second floor, it means that the observer's eye is placed at the same height: that of a man who lives on the first floor. If we reasons in metric terms, then it means five or six meters, much more than the average height of an observer placed outside the building. But we could, also, and maybe more properly, compare the height of the building with the one of the observer and we should conclude then, that the first floor is placed, more or less, at the height of the chest of who observes and that nobody could get into that flat without crawling on their hands and knees. Such a perspective gives, therefore, a sense of uncertainty and it constricts the real space.
Let's now try to lower the position of the observer, bringing it down to the height of a person who crosses the road [fig. 6]. The horizon so cuts the building a little above the half of the first level and we can appraise exactly the dimensions of Le Corbusier's beautiful architecture: the low entrance doorway (really it is about 2.4 meters high), the two superior levels marked with the brise-soleil, and the penthouse that can be seen slightly above the round building situated ahead of it, pre-existent.
This is how, in practice, the perspective incorporates the observer. But it is not all. The perspective, in fact, simulates, or better, recalls the everyday experience of the vision and we must refer to this experience in our study. For instance, why is the figure near here [fig. 7], even if including the same error as the previous figure does [fig. 5], namely a too high point of view, why is it less unpleasant?
Maybe because we are used to views of this kind, we see them continually, when we look out of our windows. This is why, wanting however to propose this image, it is better to include some foreground elements that suggests the situation in which the observer is set, the railing of a balcony, for instance.
And it seems to me, therefore, that in this shuffling of ideas and of concepts that the descriptive geometry is today, we maybe have to add to the "rules", some rules that were not at all taken into account before, simply because they were obvious, like the level of the observer in comparison to the observed object.
And so we come to the *vexata quaestio* of the "perspective distortions". The term itself, which yet is common in handbooks of all levels, leads to a sense of mistrust in the perspective, it has a negative connotation, and it should therefore be removed; also because it breeds a misunderstanding, the one relative to the permanence of these distortions and at the limit of the field angle, which would attenuate the distortions making them somehow tolerable.
It has been proved, instead, that if the eye of who observes the perspective assumes the position that was of the projection center, when the perspective was constructed, the distortions disappear, all of them and however evident they were and whatever the geometry of the picture plane is, plane or curved. So, when we talk about perspective deformations, it would always be better to use the adjective "apparent". Let's consider, to revive one of Leonardo's examples[27], a colonnade which is situated in front of the observer and which is projected onto a picture plane [fig. 8]; well, the width of the perspectives of the most peripheral columns and farthest from the observer will be greater than that of the most centrally positioned columns

[26] Every old essay and practical treatise of perspective contain these simple rules, illustrated with as many simple but efficacious drawings. What usually is not said, is that these rules help to calibrate the meter with which the objects that are represented will be measured by the eye.
[27] See Leonardo Da Vinci, Ms. A, fol. 38 recto, 1492, Paris, Institut de France.

and nearest to the observer, while, at a first sight, it should be exactly the opposite, since objects of equal greatness appear as much smaller as they are farther from who observes. But if we place our eye in a position as near as possible to what was the projection center, we will see that this distortion disappears and that the more peripheral columns seems thinner and also lower than those in the center. How can this happen?

Simply because, putting the eye in the projection center, the perspectives of the columns imply the same angles implied by the real columns in the eye of the observer, and these angles are as smaller as the columns are farther from who observes.

But, as I said, the peripheral columns will also appear lower. And this is the point, decisive according to me, that Decio Gioseffi, in his passionate critique of Panofsky, had already pointed out, but that the following literature has ignored[28]. The eye, in fact, is not at all obliged to remain "motionless", but it is free to circle around the projection center assumed as a pivot. Doing so it can observe the colonnade vanishing to the right, if looking at that side, to the left, if looking at the opposite side, and the eye can look up and also far down, namely at an inclined picture plane.

It is a simple experiment that everybody can repeat and that I already suggested some years ago in one of my writings on the perspective[29]. But if somebody would like to repeat this experiment in a much more involving way, both from a scientific point of view, and from a historical point of view, I would suggest to visit the famous corridor which leads to San'Ignazio's rooms in the Casa Professa del Gesù, corridor that, as it is well-known, has been frescoed by Andrea Pozzo. Pozzo here represented an opulent pilaster structure, corbels and beams, assuming as picture plane the walls (one of these oblique) and the vaulted ceiling, so, to start with, a picture made of several plane surfaces and curves that wrap the observer. And Pozzo believed so much in the bounded sight that the observer placed at the center of the church, free to look all around, to observe the whole solid angle of 2 Pi radians, doesn't see the slightest discontinuity, or "distortions", even if he is a fine and expert man [fig. 9]. Whereas, an observer placed far away from the center, can appreciate the incredible variety of apparent distortions, which offer a second means of understanding this work, of abstract beauty [fig. 10].

This subject is so important, not only for the perspective, but also for the history of art, that I was in doubt if I should deal with it or not, since it needs a paper of its own or maybe a whole book, and I intend, therefore, to take it up again. But here it was necessary in order to introduce the observations that will follow and to explain how, in the future of the perspective, if the perspective will have one, I also firmly hope that a response to the problem of the perspective as symbolic form can be found. If it is true, in fact, that Panofsky's essay represents a fundamental step of the artistic historiography, and I don't deny this, it is also true that the scientific elements on which it is based are totally outdated. I refer, in particular, to the knowledge of the physiology of the vision and the psychology of perception that the transactionalist school

[28] Marisa Dalai, in her commentary to Panofsky, *La questione della prospettiva*, published in Feltrinelli's edition (1973) of *Prospettiva come forma simbolica*, closes her confutation of the thesis of Gioseffi with these words: "The demonstration [of Gioseffi], detailed and very concise, does not allow a rapid explanation. But we think it is useful to mention, closing our review, the fundamental axiom: perspective picture plane and natural vision of an object coincide, giving rise to the very same retinal image – provided, "it is understood" that there is, in both cases "monocular vision and with motionless eye and from the right distance." The meaning of all the surveys and the researches mentioned as far, the sense above all of the fine studies carried out by Panofsky on the specificity of the perspective space and on the concept of space that finds expression in it, are exactly beyond that "provided." Marisa Dalai omit to mention that according to Gioseffi "the postulated immobility is not, after all, a peremptory condition nor absolutely necessary". But, having cleared this up, there remains the fact, emphasized by Gioseffi, that the scientific basis on which Panofsky's work is founded are wrong. Whether then the Panofskian thesis have a historical-critical value unrelated from these basis, and if the same basis have been nothing else than an inspiring reason, it is not up to me to judge. Certainly the confusion of different knowledges and of different ways of interpreting has brought, to the perspective, consequences whose importance still has to be valued.

[29] See O. Fasolo, R. Migliari, *Quaderni di Applicazioni della geometria descrittiva* 2, 5.10 *La sostanziale identità dei tre modelli prospettici nella veduta vincolata*, pag. 232 and following, Rome, 1983

has reversed[30]. And it is even true that some examples can be found in the history of the quadraturist painting, like the one that I mentioned, which confirm the inexistence of perspective distortions that are not only apparent. And this is, also, why the geometric study of the quadraturist painting is important.
To sum up briefly: the limits described in certain manuals with the term "visual angle" within which the perspective has to be placed in order to avoid that it deforms the represented objects, are totally arbitrary. The values that are associated to this angle are the most various and queer, and generally near to a third part of Pi; but to tell the truth, the human field angle (the eye) is near Pi and it is only limited by the prominence of the nose (Federico II from Montefeltro knew this very well, as the paintings of Piero della Francesca tell us[31]) . Moreover the perspective always warps what it represents, even in the areas near to the main point, if it is not observed correctly, namely standing in the bounded sight position. These are the reasons why, according to me, it is absurd to persist in errors that, besides all, have no comparison in the digital modelling.
Instead, it is worth to remember that the bounded sight is also the key that opens the secret messages contained in the plane anamorphosis, from Holbein's famous skull to the forbidden images in many other representations; and it is funny how these two subjects, the one that concerns the perspective and the other that concerns the anamorphosis, are generally kept separate, whilst they are applications of the same theory.
It is now necessary to draw a synthesis of what has been said so far. The man is in the center of the perspective representation both as an onlooker of the distant space (in its conceptual relationship with the infinity), and as measure of the nearer space (in its modular relationship with the architecture), and also as a protagonist of the vision (in its relationship with the projective genesis of the image).
Nowadays, the perspective doesn't have the limits anymore that it had thirty years ago, not only the conceptual limits, but also those physical. The perspective is no longer one only and static, but it can be manifold and dynamic, too. In this dynamism, as I will explain it in a little while, the three above-said bonds that man has with the space are united in an indissolubly way.
Before going into the matter, though, it is necessary to distinguish between two possible dynamics of the contemporary perspective. The first consists in the simple sequence of static images, which are showed, mostly, at twenty-five images per second, therefore a normal animation, a film, in other words. The second dynamics consists in the interactive models.
Between these to ways of using of the perspective in motion there is a profound difference. The first, in fact, is preordered and the order cannot be changed. And in this sense, even if simulating a movement, it forces the observer be in a settled position, so once again static: he enters, literally, into a perspective tunnel in which not only the movement is fixed, but also the direction of the sight. The regular movement, then, emphasizes this feeling of unnatural construction, which reduces the enjoyment of the space a lot. The observer is in practise displaced, since he observes with the eyes of another person.
In the dynamic interactive models, instead, the movement is free, as the direction of the sight is, too. The observer chooses himself where "to go", what to look at and for how long time, whether to walk or to run and at which speed, whether to stop, to stand up or to sit down and for how long time; continuing to look around. The dynamic interactive models propose, so, an experience which is similar to that of the above said corridor in Sant'Ignazio, because the space totally wraps itself around the observer who, at the same time, is free to look around as he wishes; but in this case the perspective is regenerated at every step, the picture plane is bound to the optic axis of the observer, and there is no way to reveal the illusion and the "distortion" displacing the point of sight. Some small and simple expedients, as the variation in height of the point of sight, which follows the rolling rhythm of the pace, and the possibility to bump into the objects, make the experience even more natural and convincing: we are, thanks to the perspective, immersed in the space.

[30] The results of the transactionalist school, reached in the fifties, can be found in F. P. Kilpatrick, *La psicologia transazionale*, Milan 1967. R. L. Gregory made use of these results in his famous work *Occhio e cervello, La psicologia del vedere*, published in Italian in 1966.
[31] Isn't it true that Federico, blind in the right eye, had the bridge of his nose trimmed to better see, when in battle, on that side? Or perhaps it was the result of a blow from a sword, providential though, since bereaving him of one eye, it opened, at the same time, the field of vision of the surviving eye.

As you can see, I carefully avoided using the adjective realistic and I will explain why in a while. But let's stop for a moment to consider the mentioned possibility to collide against the represented objects.
As a matter of fact, the presence of the observer, "incorporated", in this interactive perspective, is supported by the fact that this observer, known as Avatar, namely alter ego of who observes, is endowed with physical and behavioural qualities: he has a height, he has a width of shoulders, he has a body and he is subject to gravity. This brings some remarkable consequences: first of all the observer is not flying in the space, but instead he follows the ground (*follow ground* is the name of the related *behaviour*), he goes up the stairs, if there are some and he goes down a ramp, but he falls if it is too steep; furthermore, our observer cannot pass through other bodies, like walls or the glass of the windows, and he jumps slightly backwards, when it happens to him to bump into something; finally he cannot go through an opening if this is not large and wide enough.
To the measure of the space induced by the point-of-sight level that we have seen in the traditional perspective, associates here, so, a much more concrete relationship with the forms of the space. This is why, as I've already mentioned, we can properly say that the three bonds that man has with the perspective space, the idea of the infinity, the measure of the finite, and the visual participation, are united in one only harmonious emotion.
There is a last subject that I would like to touch on, regarding the interactive perspectives, and it is the one that concerns their aesthetics.
I have already spoken about a contraposition between the futile search for the photorealism in the present-day perspective and the much more arduous, but necessary, search for aesthetic values. This is more than ever true in the case of the dynamic models. It is exactly because the experience that these models propose is that of a "virtual reality", that it is necessary, I believe, to pursue the poetic transfiguration of the represented space, and then it doesn't matter if this transfiguration is achieved with the means of the photography or with those of the painting. I've got the idea that when the space to be represented is a design space, a creation, the art is the only means to represent it and to pass it on. Those who sustain the absolute impartiality of a technical drawing (and therefore don't make use of the perspective), the absolute capability of the architect to reconstruct the orthogonal projections within his mind in order to see the architecture as in a dream, they deny an incontestable fact: the always tight bonds of the architecture with the visual arts, today as in the past, and the value, also artistic, of the architectural drawing.
In this sense, the digital model of Terragni's Danteum carried out by Gabriele Pierluisi, Marco De Angelis and Luca Castagna, is an excellent example of what I have explained so far, both as regards the visual experience in itself, and as regards the aesthetics of the model, immersed in a Roman chromatism of Quaronian inspiration.

Finally I would like to mention an essential aspect of the future of the perspective: the study and the teaching that results from it; study as research, and teaching as spin-off of the research at academic formation level. The future of the perspective resides, as we have seen, in the new Humanism of the machines. This means a use of the machines not absolute, but integrated with the traditional techniques and a utilization of the same, which is not only productive, but also and above all experimental.
Having said this, in totally generic terms, the translation into facts is due to the researchers and to the students: here I cannot do much more than give some examples, like exercises assigned during the course and some laboratory experiments.
The problem of the presence of man, as measure of the space, can be faced in an essential way, representing a whole of pure and abstract volumes, at first as masses of architectural dimensions, then as forms of manual use, tools or furniture (fig. 11 and fig. 12). We are dealing with two "variations on the theme", where the theme is represented by the geometrical composition that I mentioned before and the variation is a scale variation, induced only by the presence of the observer. And it is not only a question of proportions, namely of the cut of the horizon on the represented bodies, but also of the involvement of the observer. In fact, in the case of the architecture, the man is always wrapped by the space and this condition have to be perceived, for example thanks to the vanishing of certain elements behind the shoulders of who

observes, whereas in the case of the objects, these are all situated within the visual field, "within arm's reach".
The problem of the representation of the light is not only related to the construction of the proper and the brought contours and to the measure of the light intensity degree of a plane, but, thanks to the availability of easier tools, as the rendering software are, it can be a matter of quality of the light, of its intensity and colour (both if it's morning light or day light, bright or veiled), of the depth of the horizon and the dark precipitating into the sky-blue of the zenith. Now, due to the reduced time of the teaching, which does not permit sessions *en plain air*, perhaps with a portable computer on our knees (why not?), it remains only to recover a classical exercise of the academy: to copy. So here, then, the works of the view-painters, analysed in the linear and aerial perspective, modelled in three dimensions and finally shaded by means of digital techniques, at the research, not so much of an uncritical imitation, as of a re-interpretation of the day, the hour, of the game of light and of the poetry of the moment (fig. 13 and fig. 14).
Finally it is natural to apply the experiences matured so far to the architectural model (better if projected by the student himself). And since this Model in reality is a whole of sketches, scale models, photographs, technical drawings and much more, which all converge into the project idea[32], the Model becomes the best opportunity to test the hybridisation of the techniques, namely the overlapping of semi-automatic outputs, as the rendering are, and manual interventions or, again, semi-automatic in post-production (fig. 15 and fig. 16).
Today the perspective is also, however, a laboratory suited to the research. And here I would like to give two examples: the study and the measurement of the limits of the bounded sight and the experimental verification of the continuity between plane perspective and solid perspective.
The first is a theme, in my opinion, still not explored enough. The experiment consists in the following operations:
- first of all we construct any empty space, limited by plane or curve surfaces, as one of the many rooms frescoed with trompe l'oeil perspectives could be;
- we construct an observer fixed on a point with known coordinates and we generate a perspective of the room, for instance with the optic axis perpendicular to one of the walls, the end wall (fig. 17);
- then we divide the room by means of an opaque plane, as if it was a fifth wall, perpendicular to the optic axis of the above said observer and finally we apply the perspective, already generated, onto the partition wall (fig. 18);
at this point the room gets its continuity back, since the first part, still visible, continues in the illusory part, "painted" on the wall.
We can, at this point, carry out three experiments.
The first: we verify the possibility to move the eye in all directions, moving the axis of the sight all around, noticing how the above said continuity does not stop to exist in any condition (fig. 19 and fig. 20).
The second: having made the plane perspective transparent, "painted" on the wall line-drawing, like a *sinopia*, we move the observer to a side, with small steps and we measure the corresponding displacement of the painted perspective in respect of the one observed in the real space (fig. 21).
The third: we move the observer in the primary direction, forward and backward, to observe how at every advancing corresponds a contraction of the painted space; or better this last appears shorter than the real space in proportion equal to the progression of the projection centre (fig. 22); whereas to each backward movement corresponds an equal extension of the illusory space compared with the real one.
The transparent sinopia can finally be used to verify the collimation of the perspective with the space observed in all conditions (fig. 23).
If this research was supported by a lot of experimental data on the sensibility to the displacement of a certain number of observers, I believe that we could get from these data some interesting indications about the adjustments performed by those artists, like Agostino Tassi, who are able to create whole areas of free movement of the point of sight, without loosing the illusion of the depth, as it happens, for instance, at Palazzo Lancellotti.
The second theme concerns instead a generalization of the perspective by means of the solid homology.
The digital modellers have, as it is known, some object deformation tools. These tools, called Deformers,

[32] See *Disegno come Modello*, cit.

perform twisting, tapering and other parametric transformations. It is also possible to program the transformation that you want to perform and, consequently, it is possible to program the genesis of a plane perspective and a solid perspective.
As it is well known, the solid perspective depends on the position of the observer, on the position of the plane of collineation (or plane of the trace) and of the position of the limit plane (or vanishing plane). The perspective, three-dimensional, is in this case projected into the semi-space delimited by the vanishing plane, which contains the observer. The entities, which are at an indefinite distance ahead of the observer, are projected onto the limit plane, whereas the entities that are on the plane of collineation are projected in themselves, producing perspectives coincident with the same entity. Let's now imagine to fix the position of the observer and the position of the plane of trace in respect of any architecture or part of it, like a room.
We can imagine to move the limit plane from the distant boundary of the space toward the observer, following the main direction and to assist to the forming of a solid perspective, which will imperceptibly defer from the real object during the first steps of the displacement and which will warp more and more until it assumes, finally, the perspective view defined by the position assumed by the limit plane. During this transformation the observer will not be able to perceive any change of the linear perspective of the real-space observed, whereas he can see a slow colour change of the surface, owing to the variation of the angle of incidence of the sunrays.

But we can also imagine, at this point, a translation of the plane of collineation, until it coincides with the limit plane. In this transformation, the solid perspective will be squeezed until the third dimension is effaced and until it crushes onto the picture plane.
These two passages show, therefore, the continuous transformation of the real space into a three-dimensional perspective space and the transformation of this last into a plane perspective.
The relationships that tie the coordinates of any point of the space to the coordinates of the corresponding point of the solid perspective can so be established.
Let's observe the perspective machine in a vertical section (fig. 24) and let's consider the projecting plane of any straight-line **r** (**TQ**) perpendicular to the planes π and π' (where π is the plane of collineation or of the traces and π' is the limit plane or vanishing plane). It is of little importance whether this section is vertical and in true form, or if it is in a generic position, since the relationships that we will consider do not change.
Let's call **f** the distance (**OI**'); **f** is the focal distance also called principal distance, namely the objective distance of **O** from the limit plane or the vanishing plane.
Let's, furthermore, call **d** the deepness of the space included between the two planes π and π'; **q** the length of the segment (**TQ**) of the straight-line **r** that we want to represent; and **q**' the depth of the space taken up by the image (**TQ**') of the segment (**TQ**).
The triangles (**OI'Q'**) and (**QTQ'**) are similar and we can therefore write the following relations:

q : **f** = (**TQ**') : (**I'Q**')
q' : **d** = (**TQ**') : ((**TQ**') + (**I'Q**'))

in fact **q**' is the projection of (**TQ**') on the base (**TQ**) as **d** is the projection of ((**TQ**') + (**I'Q**'));
replacing:

q' : **d** = **q** : (**q**+**f**)

therefore:

q' = **d** (**q** : (**q** + **f**)).

This relation is the base of more general relations that express the coordinates of the points of the space image **R**', in respect of the coordinates of the points of the real space **R**. In these relations the focal distance **f** and the distance **d** between the limit plane and the plane of collineation appear as constant.

Let's consider a point **P** of coordinates (**x**,**y**,**z**) and let's ask ourselves which will be the coordinates (**x'**,**y'**,**z'**) of the point **P'**, image of **P** in a solid perspective of focal length **f** and distance **d**.
The reference system has its origin in the projection centre **O**, the axis **z** coincide with the principal distance, it therefore measures the depth, the axes **x** and **y** are parallel to the above said planes, with **x** going to the right of who observes and **y** upwards.
We can write the following relations:

y' / (**y**-**y**') = **f** / **z** – (**f** - **d**);
z' / **f** = (**z** - **z**') / **z** – (**f** - **d**);

resolving:

y' = (**f y**) / (**d** + **z**) [a]
z' = (**f z**) / (**d** + **z**) [b]

As for **x**:

x' / **x** = **y**' / **y**

and, replacing:

x' / **x** = (**f y**) / (**d** + **z**) / **y**

solving:

x' = (**f x**) / (**d** + **z**) [c].

Entering the expressions [a] [b] and [c] into a mathematical Deformer[33], we obtain the solid perspective of any model, also very complex (fig. 25). The model elaborated like that can then be exported into any other modelling environment in order to obtain the metric data necessary to its realization.
Why these transformations are interesting, beyond the curiosity that they can arouse? Because they show the close analogy that ties the classic genesis of the perspective to the corresponding digital genesis. The algorithms that generate the perspective, in fact, produce a contraction of the space, exactly as in the transformations that I already showed.

Has perspective a future then?
My answer is in the above pages, but I cannot give the right answer, it will be given by the young people who today are approaching the research in the universities. This paper is dedicated and addressed to them, with the hope that it will give them a staring point for further reflections.

Rome, Wednesday 23rd February 2006

[33] For instance "Formula" in Cinema4D.

Explanations

Fig. 1 – The perspective of inclined picture plane of a parallelepiped as it appears in the most recent work of U. Saccardi, *Elementi di Proiettiva – Applicazioni della geometria descrittiva*, Florence 2004. This work is a true summa of knowledge of the descriptive geometry handed down to our time.

Fig. 2 – Again, the perspective of inclined picture plane of a parallelepiped as it was illustrated thirty years ago. The figure is taken from E. Bompiani C. Longo, *Geometria descrittiva per gli allievi di architettura*, Roma 1968. The comparison with the previous figure highlights the indifference of the mathematician for the quality of the model, or better for the capability of the perspective to evoke the forms of the space.

Fig. 3 – *Interior landscapes*, hybrid model by Gabriele Pierluisi.

Fig. 4 - *Inner city*, hybrid model by Gabriele Pierluisi.

Fig. 5 – The perspective incorporates the observer who becomes measure of the space: in this image the architecture of Le Corbusier appears smaller.

Fig. 6 – This is instead the house at La Plata in its right proportions.

Fig. 7 – The existence of a foreground that clearly puts the observer on a level higher than that of the road is the only means to justify a high position of the horizon.

Fig. 8 – Reconstruction of Leonardo's idea of the apparent distortions: the side columns have objectively larger perspectives than the ones in the centre (see segments to the right) but they are seen under smaller angles (see left) and they appear therefore thinner and also lower, as in the observation of the reality.

Fig. 9 – This strong baroque structure is not real as it look like, it is painted on the curve surface of a vault. Who find it hard to believe, can change their mind (but it is not said) observing the angels that fly among the bays.

Fig. 10 – The illusion illustrated in the preceding figure, disappears only if the observer moves significantly in respect of the projection centre: the painted forms then assume an aesthetic value completely abstract from the representation of the reality.

Fig. 11 – Exercise on the representation of pure forms like architectural volumes. Model carried out by Micaela Mattia, a student of the course *Scienza della rappresentazione II* (second year) during the academic year 2004 / 2005.

Fig. 12 – Exercise on the representation of pure forms like volumes of small dimensions. Model by Micaela Mattia.

Fig. 13 – Jean Baptiste Camille Corot, *Castel Sant'Angelo e il Tevere*, 1826 - 28. The painting is at the Louvre Museum.

Fig. 14 – Digital interpretation of the view of *Castel Sant'Angelo* painted by Corot, exercise carried out by Micaela Mattia.

Fig. 15 – The chess house: concept and drawing, using hybrid techniques, carried out by Rodolfo Migliari.

Fig. 16 - The chess house: concept and drawing, using hybrid techniques, by Rodolfo Migliari.

Fig. 17 – Digital simulation of a trompe l'oeil perspective: the real space.

Fig. 18 – Digital simulation of a trompe l'oeil perspective: the illusory space seen in bounded sight. The blade of light that falls on the wall is the only element able to reveal the illusion.

Fig. 19 – Digital simulation of a trompe l'oeil perspective: the real space as it is seen by an observer who is turning the head and eyes towards the left.

Fig. 20 – Digital simulation of a trompe l'oeil perspective: the illusory space as the same observer of the preceding figure sees it.

Fig. 21 – A sinopia painted on a transparent canvas permits to value the displacement of the observer through the comparison between the illusory space and the real space. Here the observer is moved to the left by a quantity equal to a fourth of the principal distance. The parallax, which is unperceivable as for the forms nearer to the picture, can clearly be seen in the more distant ones.

Fig. 22 – The transparent sinopia also permits to observe the relationship between the illusory space and the real space owing to a forward translation of the observer. The illusory space shrinks by a quantity equal to the forward advancing of the observer. If the observer moves forward halfway of the principal distance, the illusory space contracts in the same measure.

Fig. 23 – The transparent sinopia permits finally to observe the perfect collimation between the illusory space and the real space that can be found, in the bounded sight, in any condition.

Fig. 24 – The geometric relationships that tie, in the solid perspective, the principal distance (**f**), the distance between collineation plane and limit plane (**d**), a real depth (**q**) and its contraction in the perspective space (**q'**).

Fig. 25 – Solid perspective: the model before the distortion.

Fig. 26 – Solid perspective: the distorted model.

Fig. 27 – What the observer sees looking at the distorted model from the bounded sight.